Intermittency and taxes, what efficiency?

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1 Introduction

Electricity production from fossil fuel is one of the main causes of global warming due to green house gas emissions.[1] With the rise of social awareness and the importance given to the environment at national and European levels, this sector has attracted considerable attention in the debate on climate change mitigation. So that, several environmental policy instruments have been put in place aiming to decarbonate electricity production by reducing the share of fossil fuels in the energy mix and substituting it by renewable energy such as wind or solar power.

However, with the liberalization of the electricity market and its special specificities, this task seems more difficult. In fact, an essential feature of most renewable sources is intermittency. So in addition to difficulties of the transportation and distribution, intermittent sources raise problems at the generation stage which increases the risk of incompatibility between supply and demand on the electricity market. It is clear that the introduction of intermittent and non-storable sources of energy in the energy mix, is a new challenge for the operators and regulators of the electricity industry.

In this paper we are mainly interested in two problems of the electricity sector. The first is the efficient mix of intermittent sources (wind, solar) and conventional sources such as fossil fuel (coal, oil, natural gas). The second is to analyze the efficiency of a carbon tax to decentralize the optimal energy mix.

We are not the first to analyze the penetration of the intermittent generation technologies with perfect competition. Ambec and Crampes (2012) analyze the optimal and/or market-base provision of electricity with intermittent sources of energy. Rouillon (2015), analyze the development of the intermittent technologies given the competition of incumbent generators, under perfect and imperfect competition. Using various assumptions regarding the market power, he show that optimal policy can be decentralized under perfect competition. Twoney and Neuhoff (2009) also address the interaction between the conventional and intermittent generators. They determine the market equilibrium under perfect,

[1] A 250 MW power station operating in base (8000 hours / year) emits 1.7 Mt CO2 / year for a coal plant and 0.72 Mt CO2 / year for a gas plant.

monopolistic and duopolistic competitions. Howerer they all ignored public policies and environmental externalities.

The closest paper in the literature on public policies to decarbonate electricity production are Garcia, Alzate and Barrera (2012), and Ambec and Crampes (2015).

Garcia, Alzate and Barrera (2012) introduce RPS and FIT in a stylized model of electricity production with an intermittent source of energy. Yet they assume an inelastic demand and a regulated price cap. In contrast, price is endogenous in our paper. Our framework is more appropriate for analyzing long-term decisions concerning investment in generation capacity since in the long run smart equipment will improve demand response. It furthermore allows for welfare comparisons in which consumers' surplus and environmental damage are included.

Using a generalization of the model of Ambec and Crampes (2012), Ambec and Crampes (2015) examine the impact of public policies aiming to substitute fossil fuel by intermittent renewable sources on the energy mix. Several differences between Ambec and Crampes (2015) and the present article need to be highlighted.

Firstly, Ambec and Crampes (2015) postulate constant return to scale technologies and constant marginal damage due to pollution. Second, they assume with capacity constraints for conventional and intermittent operators. This assumption allow them to decentralize the optimal policy with a Pigovian tax. In contrast we assume an increasing marginal cost and environmental damage with no capacity constraint for the conventional sector. So far, the literature on public policies to decarbonate electricity provision has ignored the problem of intermittency. Papers have looked at pollution externalities and R D spillovers in a dynamic framework (e.g. Fischer and Newell 2008, Acemoglu et al. 2012) or in general equilibrium (Fullerton and Heutel, 2010). They have considered two technologies - a clean and a dirty one- that are imperfect substitutes in electricity production. In our paper, we are more specific about the degree of substitution: it depends on weather conditions. Consequently,capacity and production also vary with weather conditions. This introduces uncertainty in energy supply which has to be matched with a non-contingent demand. To the best of our knowledge, our paper is the first analytical assessment of public policies that deals with intermittency assuming increasing marginal damage. The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 describes the first-best energy mix. Pigovian Tax is analyzed in Section 4.In section 5 we determine an optimal tax which allow to decentralize the optimal energy mix. Finally, section 6, concludes.

2 The model

We consider a model of energy production and supply with intermittent energy. On the demand side, consumers are equipped with traditional meters, facing a flat tariff, their demand is insensitive to the short term position of the electric market. Thus, consumers sign fixed-price contracts with retailers on the forward markets. The population size is normalized to 1. Each consumer inverse demand function is $P(d)$. Define $S(d) = \int_0^d P(s)ds$ the consumer's surplus of consuming d kWh of electricity. On the supply side, electricity can be produced by means of two technologies. First, The incumbent firms supply electrical energy in quantity q using conventional generators (i.e., hydro, nuclear, coal, gas, oil). The cost function $C(q)$ represents their technology. It is assumed that $C'(0) = 0$,$C'(q) > 0$ and $C"(q) > 0$. The second technology comes from a competitive fringe, using intermittent generators (i.e., solar and wind units) who seeks to enter the market. The cost of building intermittent units with capacity $\overline{\omega}k$ is $F(k)$. It is assumed that $F(0) = 0$, $F'(k) > 0$ and $F"(k)[?]0$. The variability is modeled as a random variable ω, reflecting the climatic conditions (sun and/or wind). It is distributed on $\omega \in [w_0, \omega_1]$, with cumulative distribution function $G(\omega)$. For all ω, given the installed capacity $\overline{\omega}k$, the intermittent generation will be equal to ωk, at a negligible marginal cost. To simplify and normalize the units, it is assumed that $E[\omega] = 1$. Accordingly, the variance of ω is $V = [\omega^2] - 1 > 0$ and the intermittent generation has a capacity factor of $\frac{1}{\omega}$.

Producing electricity from conventional generators emits air pollutants (CO2) which causes damages to society. It is assumed that emissions are proportional to production. Without loss of generality, we normalize the units so that $E = q^2$. The damage from pollution depends on total pollution E. The social damage function $D(E)$ is twice continuously differentiable, increasing and (weakly) convex, i.e. $D"(E)[?]0$.[3]

Importantly, we assume that consumer demand does not vary with weather conditions. Furthermore, electricity can not be stored or transported, the only way to balance supply and demand is to rely on production adjustment or / and price variations.

For the rest of the article, we will use the following linear quadratic specification of the model:

$$P(d) = a - bd,$$

$$C(q) = \frac{1}{2}cq^2,$$

$$I(k) = \left(\gamma + \frac{1}{2}\delta k\right)k,$$

$$D(E) = e\frac{E^2}{2}$$

[2]To simplify the analysis, we assume that all fossil fuels have the same degree of emissions. i.e. we do not take into account the classification of the "merit order" of each source.

[3]With the second hypothesis, we simply forbid that the environmental damage of the last unit of pollution decreases as pollution increases, what seems to respect a general law of ecosystem functioning.

3 Optimal policy

The social problem is to choose the consumption of the consumers D, the electric generation of the conventional generators,$q(\omega)$, for all ω, the capacity of the intermittent generators, k, and emissions, $E(\omega)$,for all ω, to maximize:

$$\int_{\omega_0}^{\omega_1} [S(D) - C(q(\omega)) - F(k) - D(E(\omega))]\, dG(\omega)$$

subject to

$$D = q(\omega) + \omega k$$

and

$$E(\omega) = q(\omega)$$

for all ω

The first constraint is the non-reactivity constraint. It implies that the supply of electricity (conventional and intermittent) must bind consumer demand. The second represents the relation between the production of electricity from fossils fuels and CO2 emissions.

Integrating the second constraint, the Lagrangian of this problem writes:

$$L = \int_{\omega_0}^{\omega_1} \left[\begin{array}{c} S(D) - C(q(\omega)) - F(k) - D(q(\omega)) \\ -\lambda(\omega)(D - q(\omega) - \omega k) \end{array} \right] dG(\omega)$$

Where the Lagrangian multiplier $\lambda(\omega)$ reflects the implicit price of electricity in the state ω.[4]

Let D^0, $q^0(\omega)$ and k^0 be the solution. It satisfies the first order conditions:

$$P\left(D^0\right) = \int_{\omega_0}^{\omega_1} (C'(q^0(\omega)) + D'(q^0(\omega)))dG(\omega)$$

$$F'\left(k^0\right) = \int_{\omega_0}^{\omega_1} (C'(q^0(\omega)) + D'(q^0(\omega)))\omega dG(\omega)$$

The above conditions and solutions have natural economic interpretations. Integrating environmental damage in the analysis increases the cost of conventional operators. In fact, the latter includes not only operating costs $C'(q(\omega))$ but also environmental costs $D'(E(\omega))$.

In words, the consumers should raise their consumption as long as their marginal propensity to pay is larger than the expected (implicit) price of electricity. The conventional generators should increase their production as long as their marginal cost is smaller than the (implicit) price of electricity minus the marginal damage of the environment . The intermittent generators should increase their capacity as long as their marginal cost of investment is smaller

[4] Below, it will be assumed that $\lambda(\omega) > 0$, for all ω. This assumption is reasonable most of the time and simplifies the presentation.However, note that because of the negligible marginal costs of renewable energy, the electricity prices on the spot market can sometimes be negative. In Germany this phenomenon occurred 15 times in 2011 (Benhmad and Percebois 2013

than their expected marginal benefit of investment. In the state of nature ω, the marginal benefit of investing in the intermittent units is the product of the implicit price of electricity times the productivity of the marginal generating unit ω.

Considering the linear-quadratic specification, from (9) to (10), we can show :

$$q^0(\omega) = \frac{a - bk\omega}{b + c + e}$$

$$k^0 = \frac{a\frac{c+e}{b+c+e} - \gamma}{\delta + b\frac{c+e}{b+c+e}\,(V+1)}$$

$$D^0 = \frac{a + \frac{a\frac{c+e}{b+c+e} - \gamma}{\delta + b\frac{c+e}{b+c+e}(V+1)}\,(c+e)}{b + c + e}$$

4 Perfect competition

In this section, assuming perfect competition, we analyze the impact of a tax on pollutants emitted by conventional generators on electricity production and welfare. We consider a market economy with free entry and price-taker producers and retailers.

The system of markets is as follows, there is a full set of spot market and forward markets. On the spot market electricity operators sell to retailers at real-time pricing. Consumers sign forward contracts with retailers at fixed prices.[5]

Finally, assume that the regulator in charge of general interest charges a tax to conventional generators on their polluting emitted. It is assumed that this tax is proportional to emissions. Let T denote the tax rate per unit of emissions.

The timing of the decisions is the following. In the first stage, the intermittent generators invest in generating units (k). In the second stage,consumers sign contracts with retailers (quantities D and $\overline{q}$ at price $\overline{p}$). In the third stage nature determines the climatic conditions. In the last stage, generating units decide their electricity production on the spot market ($q\left(\omega\right)$ and $r\left(\omega\right)$ at price $p\left(\omega\right)$)

The market equilibrium is now obtained by working backward in the game tree.

4.1 Spot market

Let $\overline{p}$ and $\overline{q}$ respectively be the equilibrium price and the volume of contract that retailers have signed on the forward market . For all ω, let $p\left(\omega\right)$ represent the equilibrium spot price of electricity in the state ω. The intermittent generators supply ωk. [6]. The spot market clearing condition is: $\overline{q} = q\left(\omega\right) + \omega k$

[5]Retailers are risk neutral

[6]They choose $r\left(\omega\right) \leq \omega k$ to maximize: $\pi = p(\omega)r(\omega)$

Conventional generators supply $q(\omega)$ such that[7]

$$C'(q(\omega)) + T = p(\omega)$$

This condition means that the conventional operators increase their production as long as their marginal cost, including the tax, are smaller than the electricity market price. The market price is determined through the marginal costs of conventional operators. The introduction of the tax increase their operating costs. These extra costs will be charged to the retailers, increasing the electricity price on the spot market. Retailers in their turn, will charge these additional costs to final consumers on the forward market.

4.2 Forward market

Consider the market of contracts. Each consumer demands D such that $P(D) = \overline{p}$. Retailers supply $\overline{q}$ at price $\overline{p}$. They anticipate they will buy their electricity at the spot price $p(\omega)$, for all ω. Thus, in equilibrium, the price of contracts $\overline{p}$ must be equal to the expected price of the electricity on the spot markets $\overline{p} = E[p(\omega)]$. The forward clearing condition is $D = \overline{q}$.

The equilibrium forward market checks:

$$P(D) = \int_{\omega_0}^{\omega_1} p(\omega)\, dG(\omega)$$

This condition means that consumers should raise their consumption as long as their marginal propensity to pay is larger than the expected price of electricity[8].

4.3 Investment

The intermittent generators anticipate the equilibrium price $p(\omega)$, for all ω and correspondingly choose k to maximize:

$$\pi = \int_{\omega_0}^{\omega_1} (p(\omega)\,\omega k) dG(\omega) - F(k)$$

Under the assumption of perfect competition, the equilibrium capacity will satisfy:

$$F^{'}(k) = \int_{\omega_0}^{\omega_1} p(\omega)\omega dG(\omega)$$

This condition means that the intermittent generators should increase their capacity as long as their marginal cost of investment is smaller than their expected marginal profit of investment[9].This equality shows that intermittent operators benefit from the introduction of the tax due to a higher market price which induce an increase of their installed capacities.

[7] They choose $q(\omega)$ to maximize :$\pi = p(\omega)q(\omega) - C(q(\omega)) - Tq(\omega)$

[8] Using $E[\omega] = 1$, we have $\int_{\omega_0}^{\omega_1} p(\omega)\, dG(\omega) = \frac{Tb+c(a-bk)}{b+c}$

[9] Using $E\left[\omega^2\right] = (V+1)$, we have $\int_{\omega_0}^{\omega_1} p(\omega)\,\omega dG(\omega) = \frac{Tb+c(a-bk(V+1))}{b+c}$

4.4 Equilibrium outcome

Considering the linear-quadratic specification, from (15) to (16), we can show that the equilibrium outcome (denoted $q^*(\omega)$, for all (ω),D^* and k^*) is:

$$q^*(\omega) = \frac{a - bk\omega - T}{b + c}$$

$$p^*(\omega) = \frac{Tb + c(a - bk\omega)}{b + c}$$

$$D^* = \frac{c(a-\gamma) - (T-a)\left(\frac{Vbc}{(b+c)} + \delta\right)}{\delta(b+c) + bc(V+1)}$$

$$P(D^*) = \frac{bc\gamma + \left(\delta + \frac{Vbc}{(b+c)}\right)(Tb + ac)}{\delta(b+c) + bc(V+1)}$$

$$k^* = \frac{\frac{Tb+ac}{b+c} - \gamma}{\delta + bc\frac{V+1}{b+c}}$$

To summarize, the tax has mainly three effects. First, it increases the marginal costs of conventional generators,following that, their electricity production $q^*(\omega)$, decline.[10] Second, These extra costs will be charged to the final consumer through retailers which increase the electricity prices on the forward market $P(D^*)$ resulting on the reduction the electricity consumption D^*. [11]. Finally, Intermittent operators, benefit when to them, from a higher selling price, which results in the increase of the installed capacity intermittent (k).[12].

To decentralize the social optimum,i.e. so that equations (9) and (14) coincide, the tax rate should be equal to the environmental marginal damage, in every state of the world. Formally $T = D'(E(\omega))$. However in our model the marginal damage depends on weather conditions which prevents the Pigovian tax to decentralize the social optimum[13].

5 Optimal tax:

The introduction of intermittent sources of energy in the energy mix, is a new challenge for the operators and regulators of the electricity industry. Regulators must adjust policy instruments to deal with intermittency and to achieve environmental goals. Using the previous results we determine the tax rate which decentralize the optimal state.

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[10] We can show that: $\frac{\partial}{\partial T}(q^*(\omega)) = -\frac{1}{b+c} < 0$.

[11] We can show that: $\frac{\partial}{\partial T}(P(D^*)) = b\frac{b\delta + c\delta + Vbc}{(b+c)(b\delta + c\delta + bc + Vbc)} > 0$ and $\frac{\partial}{\partial T}(D^*) = -\frac{b\delta + c\delta + Vbc}{(b+c)(b\delta + c\delta + bc + Vbc)} < 0$

[12] We can show that: $\frac{\partial}{\partial T}(k^*) = \frac{b}{b\delta + c\delta + bc + Vbc} > 0$

[13] In fact usually the rate tax is set every year at a fixed and known rate

The social problem is to choose the consumption of the consumers D^t, the electric generation of the conventional generators, $q^t(\omega)$, for all ω , the capacity of the intermittent generators,k^t,emissions, $E(\omega)$),for all (ω), and the tax T^* to maximize:

$$\int_{\omega_0}^{\omega_1} \left[S(D) - C\left(q\left(\omega\right)\right) - F\left(k\right) - D(E\left(\omega\right))\right] dG(\omega)$$

subject to :

$$D = q\left(\omega\right) + \omega k$$

$$E\left(\omega\right) = q\left(\omega\right)$$

$$P(D) = \int_{\omega_0}^{\omega_1} C'\left(q\left(\omega\right)\right) dG(\omega) + T$$

$$F^{'}(k) = \int_{\omega_0}^{\omega_1} C'\left(q\left(\omega\right)\right) \omega dG(\omega) + T$$

For all ω.

Integrating the second constraint, the Lagrangian of this problem writes:

$$L = \int_{\omega_0}^{\omega_1} \left[\begin{array}{c} S(D) - C\left(q\left(\omega\right)\right) - F\left(k\right) - D(q\left(\omega\right)) \\ -\lambda\left(\omega\right)\left(D - q\left(\omega\right) - \omega k\right) \\ -\phi\left(\omega\right)\left(P(D) - C'\left(q\left(\omega\right)\right) - T\right) \\ -\beta\left(\omega\right)\left(F^{'}(k) - C'\left(q\left(\omega\right)\right)\omega - T\omega\right) \end{array} \right] dG(\omega)$$

Where the Lagrangian multiplier $\lambda\left(\omega\right)$ reflects the implicit price of electricity in the state ω.

Let D^t,$q^t\left(\omega\right)$ and k^t and T^* be the solution. It satisfies the following first order conditions:

$$\lambda\left(\omega\right) = D^{'}\left(q\left(\omega\right)\right) + C^{'}\left(q\left(\omega\right)\right) - \beta\left(\omega - 1\right) C^{''}\left(q\left(\omega\right)\right)$$

$$P(D) + \beta P^{'}(D) = \int_{\omega_0}^{\omega_1} \lambda\left(\omega\right) dG(\omega)$$

$$F^{'}\left(k\right) + \beta F^{''}\left(k\right) = \int_{\omega_0}^{\omega_1} \lambda\left(\omega\right) \omega dG(\omega)$$

Eliminating multiplier we get:

$$P(D^t) = \int_{\omega_0}^{\omega_1} \left(C^{'}\left(q\left(\omega\right)\right) + D^{'}(q\left(\omega\right)) \frac{F^{''}(k) - P^{'}(D)\omega}{C''\left(q\left(\omega\right)\right)\left(\omega - 1\right)^2 + F''(k) - P'(D)}\right) dG(\omega)$$

$$F^{'}\left(k^t\right) = \int_{\omega_0}^{\omega_1} \left(C^{'}\left(q\left(\omega\right)\right)\omega + D^{'}(q\left(\omega\right)) \frac{F^{''}(k) - P^{'}(D)\omega}{C''\left(q\left(\omega\right)\right)\left(\omega - 1\right)^2 + F''(k) - P'(D)}\right) dG(\omega)$$

The optimal tax rate:

$$T^{*} = \int_{\omega_0}^{\omega_1} D^{'}(q(\omega)) \frac{F^{''}(k) - P^{'}(D)\omega}{C^{''}(q(\omega))(\omega-1)^2 + F^{''}(k) - P^{'}(D)} dG(\omega)$$

5.1 Discussion

These results are different from what we have expected. In fact, it was expected that the tax rate should be equal to the average of the marginal environmental damage, i.e. T = D (E (w)). However, analysis shows that the optimal tax rate which decentralize the optimal state, depends not only on the marginal environmental damage but also on intermittent capacity, the availability of these sources and the willingness of consumers to pay for electricity. So the bigger the amount of installed intermittent capacity is, more often these sources are available and more consumers are reactive to price, the lower the tax rate should be.

These results have natural economic interpretations. First,the tax aims to reduce the negative externalities of electricity production from fossil fuel. So if the share of intermittent capacities increase in the energy mix, less we will have to correct externalities.

Second,in our setting, apart pollution externalities, the regulator should also internalize another externalities which is the non reactivity of consumers. So the tax should take into account both of these two externalities to balance their effects.

Thus, if we compare the results of the optimal tax with those in optimal state (Equations (33) and (34) with equations (9) and (10)), we can see that the optimal tax increases both of the market price and intermittent capacity above the optimal level. This result is interesting and suggests that internalizing the negative externalities of fossil production on the environment requires an over investment in intermittent to balance the effects of the externalities caused by the non reactive consumers.

These results are different from those of the optimal state because the latter deal only with environmental damage and ignores the supply sides' externalities.

Finally, the effect of the increase of the tax rate on the total electricity production comes from two factors. First the tax increase the operational costs of conventional operators [14]which makes intermittent generators more competitive. This leads to the increase of the installed intermittent capacities[15]. Second, following the increase of conventional operators operational costs, consumers face higher market price[16]. Thus, the total installed capacities depends on the reactivity of consumers to the changes of the retail price[17]. The lower is the elasticity of the demand the bigger the total electricity production would be.

[14] $\frac{\partial}{\partial T} q^t(\omega) = \frac{-1}{(b+c)}$

[15] $\frac{\partial}{\partial T} k^t = \frac{b}{\delta(b+c)+bc(V+1)}$

[16] $\frac{\partial}{\partial T} P(D^t) = b\frac{\delta(b+c)+Vbc}{(b+c)(\delta(b+c)+bc(V+1))}$

[17] $\frac{\partial}{\partial T}\left(q^t(\omega) + k^t\right) = \frac{b(b-Vc)-\delta(b+c)}{(b+c)(\delta(b+c)+bc(V+1))}$

6 Appendix

6.1 Social optimum

Considering the linear quadratic specification of the model. The optimal allocation (D^0,$q^0(\omega)$ for allω and k^0) satisfies the following system:

$$D^0 = q^0(\omega) + \omega k$$

$$a - bD^0 = (e+c)\int_{\omega_0}^{\omega_1} q^0(\omega)\, dG(\omega)$$

$$\gamma + \delta k^0 = (e+c)\int_{\omega_0}^{\omega_1} q^0(\omega)\,\omega dG(\omega)$$

Using the two first equations to calculate:

$$q^0(\omega) = \frac{a - bk\omega}{b+c+e}$$

Then substitute into the last two equations and integrate (using $E[\omega] = 1$ and $E\left[\omega^2\right] = V + 1$) to write

$$a - bD^0 = (e+c)\frac{a-bk}{b+c+e}$$

$$\gamma + \delta k^0 = (e+c)\frac{a - bk(V+1)}{b+c+e}$$

Finally solve this system to obtain:

$$D^0 = \frac{a + \frac{a\frac{c+e}{b+c+e} - \gamma}{\delta + b\frac{c+e}{b+c+e}(V+1)}(c+e)}{b+c+e}$$

$$k^0 = \frac{a\frac{c+e}{b+c+e} - \gamma}{\delta + b\frac{c+e}{b+c+e}(V+1)}$$

6.2 Optimal tax:

Integrate (26) and (27) into (30) and (31) and solving the system we get:

$$\int_{\omega_0}^{\omega_1} \beta(\omega) = \frac{P^{'}(D)D^{'}(q(\omega))\omega - T\left(C^{''}(q(\omega))(\omega-1) + P^{'}(D)\right)}{C^{''}(q(\omega))P^{'}(D)\omega^2 - F^{''}(k)\left(C^{''}(q(\omega)) - P^{'}(D)\right)} dG(\omega)$$

Then substitute into equation (32)to write:

$\int_{\omega_0}^{\omega_1}\left(\frac{F^{''}(k)D^{'}(q(\omega)) - T\left(F^{''}(k) + C^{''}(q(\omega))\omega(\omega-1)\right)}{C^{''}(q(\omega))\left(F^{''}(k) - P^{'}(D)\omega^2\right) - F^{''}(k)P^{'}(D)} + \omega\frac{P^{'}(D)\omega D - T\left(P^{'}(D) + C^{''}(q(\omega))(\omega-1)\right)}{F^{''}(k)P^{'}(D) - C^{''}(q(\omega))\left(F^{''}(k) - P^{'}(D)\omega^2\right)}\right) dG(\omega) =$
0

Solving it for the tax rate we get:

$$T^{*}=\int_{\omega_0}^{\omega_1} D^{'}\left(q\left(\omega\right)\right)\frac{F^{''}(k)-P^{'}(D)\omega^{2}}{F^{''}(k)-P^{'}(D)\omega}dG(\omega)$$

Then substitute T , $\phi\left(\omega\right)$ and $\beta\left(\omega\right)$ into equation (29) to write:

$$\int_{\omega_0}^{\omega_1}\lambda\left(\omega\right)=\int_{\omega_0}^{\omega_1}\left(C^{'}\left(q\left(\omega\right)\right)+D^{'}\left(q\left(\omega\right)\right)-C^{''}\left(q\left(\omega\right)\right)\frac{T\left(F^{''}(k)-P^{'}(D)\omega\right)-D^{'}(q(\omega))\left(F^{''}(k)-P^{'}(D)\omega^{2}\right)}{C^{''}(q(\omega))\left(F^{''}(k)-P^{'}(D)\omega^{2}\right)-F^{''}(k)P^{'}(D)}\right)dG(\omega)$$

Finally substituting into equations (30) and (31) to write :

$$P(D^{t})=\int_{\omega_0}^{\omega_1}\left(C^{'}\left(q\left(\omega\right)\right)+D^{'}\left(q\left(\omega\right)\right)\frac{F^{''}(k)-P^{'}(D)\omega^{2}}{F^{''}(k)-P^{'}(D)\omega}\right)dG(\omega)$$

$$F^{'}\left(k^{t}\right)=\int_{\omega_0}^{\omega_1}\left(C^{'}\left(q\left(\omega\right)\right)\omega+D^{'}\left(q\left(\omega\right)\right)\frac{F^{''}(k)-P^{'}(D)\omega^{2}}{F^{''}(k)-P^{'}(D)\omega}\right)dG(\omega)$$

Considering the linear quadratic specification of the model. The optimal allocation (D^{t},$q^{t}\left(\omega\right)$for allω and k^{t}) satisfies the following system:

$$D^{t}=q^{t}\left(\omega\right)+\omega k$$

$$P(D^{t})=\int_{\omega_0}^{\omega_1} C^{'}\left(q\left(\omega\right)\right)dG(\omega)+T$$

$$F^{'}\left(k^{t}\right)=\int_{\omega_0}^{\omega_1} C^{'}\left(q\left(\omega\right)\right)\omega dG(\omega)+T$$

Using the two first equations to calculate:

$$q^{t}\left(\omega\right)=\frac{a-bk\omega-T}{b+c}$$

Then substitute into the last two equations and intergrate (using $E\left[\omega\right]=1$ and $E\left[\omega^{2}\right]=V+1$) to write

$$a-bD^{t}=c\frac{a-bk-T}{b+c}+T$$

$$\gamma+\delta k^{t}=c\frac{a-bk\left(V+1\right)-T}{b+c}+T$$

Finally solve this system to obtain:

$$D^{t}=\frac{c\left(a-\gamma\right)-\left(T-a\right)\frac{b\delta+c\delta+Vbc}{b+c}}{\delta\left(b+c\right)+bc\left(V+1\right)}$$

$$k^{t}=\frac{c\left(a-\gamma\right)-b\left(\gamma-T\right)}{\delta\left(b+c\right)+bc\left(V+1\right)}$$

The tax effect on output, price and demand:

$\frac{\partial}{\partial T}D^{t}=-\frac{\delta(b+c)+Vbc}{(b+c)(\delta(b+c)+bc(V+1))}$

$\frac{\partial}{\partial T}k^{t}=\frac{b}{\delta(b+c)+bc(V+1)}$

$\frac{\partial}{\partial T}P(D^{t})=b\frac{\delta(b+c)+Vbc}{(b+c)(\delta(b+c)+bc(V+1))}$

$\frac{\partial}{\partial T}\left(q^{t}\left(\omega\right)+k^{t}\right)=\frac{b(b-Vc)-\delta(b+c)}{(b+c)(\delta(b+c)+bc(V+1))}$

7 References

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